



UNIVERSITY OF SASKATCHEWAN
DEPARTMENT OF MATHEMATICS & STATISTICS
MATH. 224.3 (ALL SECTIONS)

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Midterm Examination #1

Time: 4:30-5:50

IMPORTANT

- Print your name and encode your student number on the multiple choice sheet.
- Open Book Examination: Students may use "Calculus" by J. Stewart and notes distributed by the instructors.
- No Calculators or formula sheets are allowed.
- All questions are of equal value.

1. All the 5th roots of $16\sqrt{2} + 16\sqrt{2} \cdot i$ lie on a circle in the complex plane. What is the area of this circle?

(A) 64π (B) $32\sqrt{2}\pi$ (C) 32π (D) $16\sqrt{2}\pi$ (E) 16π (F) 8π (G) 4π (H) 2π (I) π (J) $\frac{\pi}{2}$

2. If $y = y(x)$ is the solution of the IVP $\frac{y' - e^{-x} + 2}{y} = -2$, $y(0) = -2$, then $y(\ln 2^{-1})$ equals to:

(A) -4 (B) -5 (C) -6 (D) -7 (E) -8 (F) -9 (G) -10 (H) -11 (I) -12 (J) -13

3. Find the orthogonal trajectories of the family of curves $y = \frac{k}{x^3}$, where k is an arbitrary constant.

(A) $y^2 - 7x^2 = C$ (B) $y^2 - 6x^2 = C$ (C) $y^2 - 5x^2 = C$

(D) $y^2 - 4x^2 = C$ (E) $y^2 - 3x^2 = C$ (F) $y^2 - 2x^2 = C$

(G) $y^2 - x^2 = C$ (H) $2y^2 - x^2 = C$ (I) $3y^2 - x^2 = C$ (J) $4y^2 - x^2 = C$

4. An object is taken from an oven when its temperature is 200°C and is placed in a second oven where the temperature is kept at a constant 40°C . At 5:00 p.m. the temperature of the object is 120°C and two hours later it is 80°C . At what time was the object taken from the first oven?

(A) 4:00 p.m. (B) 3:30 p.m. (C) 3:00 p.m. (D) 2:30 p.m. (E) 2:00 p.m.
 (F) 1:30 p.m. (G) 1:00 p.m. (H) 12:30 p.m. (I) 12:00 p.m. (J) 11:30 a.m.

5. Solve the second-order DE $y'' + y' - 6y = e^{-3x}$ by using the method of undetermined coefficients.

(A) $y = c_1 e^{2x} + c_2 e^{-3x} + \frac{1}{5}x$ (B) $y = c_1 e^{2x} + c_2 x e^{-3x} + \frac{2}{5}x e^{-3x}$
 (C) $y = c_1 e^{2x} + c_2 e^{-3x} + \frac{1}{3}x e^{-3x}$ (D) $y = c_1 e^{2x} + c_2 e^{-3x} - \frac{1}{3}x e^{-3x}$
 (E) $y = c_1 e^{2x} + c_2 e^{-3x} + \frac{3}{5}x e^{-3x}$ (F) $y = c_1 e^{2x} + c_2 e^{-3x} - \frac{1}{5}x e^{-3x}$
 (G) $y = c_1 e^{2x} + c_2 e^{-3x} + \frac{1}{7}x e^{-3x}$ (H) $y = c_1 x e^{2x} + c_2 x e^{-3x} - \frac{1}{7}x e^{-3x}$
 (I) $y = c_1 e^{2x} + c_2 e^{-3x} + x e^{-2x}$ (J) $y = c_1 e^{2x} + c_2 e^{-3x} - x e^{-2x}$

Questions 6 and 7 deal with the second-order differential equation $y'' - 2y' + y = \frac{e^x}{x^2}$.

6. The general solution y_c of the complementary equation of the above DE is:

(A) $c_1 e^x + c_2 x e^x$ (B) $c_1 e^{2x} + c_2 x e^x$ (C) $c_1 e^x + c_2 x e^{2x}$ (D) $c_1 \frac{1}{x} e^x + c_2 x e^x$
 (E) $c_1 e^x + c_2 \frac{1}{x^2} e^x$ (F) $c_1 e^x + c_2 x^3 e^x$ (G) $c_1 e^x + c_2 x e^{3x}$ (H) $c_1 e^{3x} + c_2 x e^x$
 (I) $c_1 e^x + c_2 x e^{4x}$ (J) $c_1 e^{4x} + c_2 x e^x$

7. A particular solution y_p of the above DE is:

(A) $\frac{e^x}{1 + \ln x}$ (B) $\frac{1 + \ln x}{e^x}$ (C) $-\frac{e^x}{1 + \ln x}$ (D) $-\frac{1 + \ln x}{e^x}$ (E) $e^x(1 + \ln x)$
 (F) $-e^x(1 + \ln x)$ (G) $-e^x(1 + \ln x^2)$ (H) $-e^x(1 + \ln x^3)$ (I) $-e^x(1 + \ln x^{-2})$
 (J) $-e^x(1 + \ln x^{-3})$

8. Find $y'(0)$, where $y(x)$ denotes the solution of the boundary value problem $y'' - 16y = 0$, $y(0) = 0$, $y\left(\frac{1}{4}\right) = 1$.

(A) $\frac{8e}{e^2 - 1}$ (B) $\frac{7e}{e^2 - 1}$ (C) $\frac{6e}{e^2 - 1}$ (D) $\frac{5e}{e^2 - 1}$ (E) $\frac{4e}{e^2 - 1}$
 (F) $\frac{3e}{e^2 - 1}$ (G) $\frac{e}{e^2 - 1}$ (H) $\frac{e}{e^2 + 1}$ (I) 0 (J) 1

9. General solution of the DE $\frac{dy}{dx} = \frac{x^3y}{x^4 + y^4}$, $(x, y) \neq (0, 0)$, is implicitly given by:

(A) $4 \ln |y| = \left(\frac{x}{y}\right)^4 + C$ (B) $3 \ln |y| = \left(\frac{y}{x}\right)^3 + C$ (C) $y = \left(\frac{y}{x}\right)^4 + C$
 (D) $4 \ln |y| + x = \left(\frac{x}{y}\right)^4 + C$ (E) $3 \ln |y| + \frac{1}{x} = \left(\frac{y}{x}\right)^3 + C$ (F) $e^y = \left(\frac{y}{x}\right)^4 + x$
 (G) $xe^y = \left(\frac{y}{x}\right)^3 + xy$ (H) $4 \ln |y| = \frac{x^3}{y^4} + C$ (I) $3 \ln |y| = \frac{y^4}{x^3} + C$

10. A tank initially contains 1000 gal of water in which is dissolved 20 gal of salt. A valve is opened and water containing 0.2 gal of salt per gallon flows into the tank at a rate of 5 gal/min. The mixture in the tank is well stirred and drains from the tank at a rate of 5 gal/min. What would be the amount of salt in the tank after a very long period of time?

(A) 1000 (B) 900 (C) 800 (D) 700 (E) 600 (F) 500 (G) 400 (H) 300 (I) 200 (J) 100

THE END